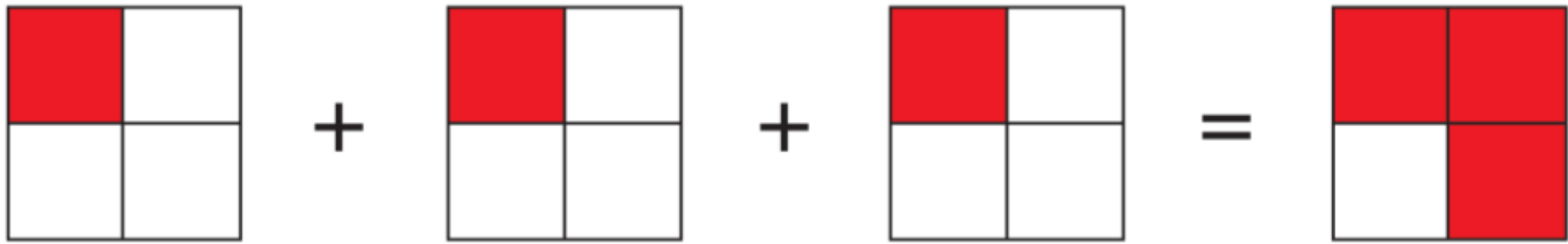MARIA
MATH TEACHER
PRESIDENT

A GIFT

just for you

YOU CAN EXCEL AT MATHEMATICS

JUAN
MATH TUTOR
VICE-PRESIDENT

Multiplying Fractions by Whole Numbers

Recall that multiplication by a whole number can be represented as repeated addition. For example, $4 \cdot 5 = 5 + 5 + 5 + 5$. You can multiply a whole number by a fraction using the same method.

$$3 \cdot \frac{1}{4} \ = \ \frac{1}{4} \ + \ \frac{1}{4} \ + \ \frac{1}{4} \ = \ \frac{3}{4}$$

There is another way to multiply with fractions. Remember that a whole number can be written as an improper fraction with 1 in the denominator. So $3 = \frac{3}{1}$.

$$\frac{3}{1} \cdot \frac{1}{4} = \frac{3 \cdot 1}{1 \cdot 4} = \frac{3}{4}$$

⟵ **Multiply numerators.**
⟵ **Multiply denominators.**

Notice that when you multiply a proper fraction and a number greater than 1, the product is between the factors.

E X A M P L E 1 Multiplying Fractions and Whole Numbers

Multiply. Write each answer in simplest form.

Method 1: Use repeated addition.

A $5 \cdot \frac{1}{8}$

$5 \cdot \frac{1}{8} = \frac{1}{8} + \frac{1}{8} + \frac{1}{8} + \frac{1}{8} + \frac{1}{8}$ *Write $5 \cdot \frac{1}{8}$ as addition. Add the numerators.*

$= \frac{5}{8}$

B $3 \cdot \frac{1}{9}$

$3 \cdot \frac{1}{9} = \frac{1}{9} + \frac{1}{9} + \frac{1}{9}$ *Write $3 \cdot \frac{1}{9}$ as addition. Add the numerators.*

$= \frac{3}{9}$

$= \frac{1}{3}$ *Write your answer in simplest form.*

Method 2: Multiply.

C $4 \cdot \frac{7}{8}$

$\frac{4}{1} \cdot \frac{7}{8} = \frac{28}{8}$ *Multiply.*

$= \frac{7}{2}$ or $3\frac{1}{2}$ *Write your answer in simplest form.*

EXAMPLE 2

Evaluating Fraction Expressions

Evaluate 6x for each value of x. Write each answer in simplest form.

A $x = \frac{1}{8}$

$6x$	*Write the expression.*
$6 \cdot \frac{1}{8}$	*Substitute $\frac{1}{8}$ for x.*
$\frac{6}{1} \cdot \frac{1}{8} = \frac{6}{8}$	*Multiply.*
$= \frac{3}{4}$	*Write your answer in simplest form.*

B $x = \frac{2}{3}$

$6x$	*Write the expression.*
$6 \cdot \frac{2}{3}$	*Substitute $\frac{2}{3}$ for x.*
$\frac{6}{1} \cdot \frac{2}{3} = \frac{12}{3}$	*Multiply.*
$= \frac{4}{1}$	
$= 4$	

Sometimes the denominator of an improper fraction will divide into the numerator without a remainder, as in Example 2B. When this happens, the improper fraction is equivalent to a whole number, not a mixed number.

$$\frac{12}{3} = 4$$

EXAMPLE 3

Social Studies Application

Any proposed amendment to the U.S. Constitution must be ratified, or approved, by $\frac{3}{4}$ of the states. When the 13th Amendment abolishing slavery was proposed in 1865, there were 36 states. How many states needed to ratify this amendment in order for it to pass?

To find $\frac{3}{4}$ of 36, multiply.

$$\frac{3}{4} \cdot 36 = \frac{3}{4} \cdot \frac{36}{1}$$

$$= \frac{108}{4} \qquad \textit{Divide 108 by 4 and write your answer}$$

$$= 27 \qquad \textit{in simplest form.}$$

For the 13th Amendment to pass, 27 states had to ratify it.

Multiplying Fractions

Your brain keeps working even when you're asleep. It makes sure that you keep breathing and that your heart keeps beating.

On average, people spend $\frac{1}{3}$ of their lives asleep. About $\frac{1}{4}$ of the time they sleep, they dream. What fraction of a lifetime does a person typically spend dreaming?

One way to find $\frac{1}{4}$ of $\frac{1}{3}$ is to make a model.

$$\frac{1}{4} \cdot \frac{1}{3} = \frac{1}{12}$$

Notice that there is 1 section where the shading overlaps, and the product of the numerators is 1. Notice that there are 12 sections in the model, and the product of the denominators is 12. You can use these observations to write a rule for multiplying fractions.

$\frac{1}{4} \cdot \frac{1}{3} = \frac{1 \cdot 1}{4 \cdot 3}$ ⟵ *Multiply the numerators.*
                        ⟵ *Multiply the denominators.*

$= \frac{1}{12}$ *The answer is in simplest form.*

A person typically spends $\frac{1}{12}$ of his or her lifetime dreaming.

Multiplying Fractions

Multiply. Write each answer in simplest form.

A $\frac{1}{3} \cdot \frac{3}{5}$

$\frac{1}{3} \cdot \frac{3}{5} = \frac{1 \cdot 3}{3 \cdot 5}$ *Multiply numerators. Multiply denominators.*

$= \frac{3}{15}$ *The GCF of 3 and 15 is 3.*

$= \frac{1}{5}$ *The answer is in simplest form.*

B $\frac{6}{7} \cdot \frac{2}{3}$

$\overset{2}{\cancel{6}}{7} \cdot \frac{2}{\underset{1}{\cancel{3}}} = \frac{2}{7} \cdot \frac{2}{1}$ *Use the GCF to simplify the fractions before multiplying. The GCF of 6 and 3 is 3.*

$= \frac{2 \cdot 2}{7 \cdot 1}$ *Multiply numerators. Multiply denominators.*

$= \frac{4}{7}$ *The answer is in simplest form.*

C $a = \frac{3}{4}$ $\qquad a \cdot \frac{1}{3}$

$\qquad\qquad\quad \frac{3}{4} \cdot \frac{1}{3}$ $\qquad$ *Substitute $\frac{3}{4}$ for a.*

$\qquad\qquad\quad \frac{3 \cdot 1}{4 \cdot 3}$ $\qquad$ *Multiply numerators. Multiply denominators.*

$\qquad\qquad\quad \frac{3}{12}$ $\qquad$ *The GCF of 3 and 12 is 3.*

$\qquad\qquad\quad \frac{1}{4}$ $\qquad$ *The answer is in simplest form.*

EXAMPLE 2 **Evaluating Fraction Expressions**

Evaluate the expression $a \cdot \frac{1}{3}$ for each value of *a*. Write the answer in simplest form.

A $a = \frac{5}{8}$ $\qquad a \cdot \frac{1}{3}$

$\qquad\qquad\quad \frac{5}{8} \cdot \frac{1}{3}$ $\qquad$ *Substitute $\frac{5}{8}$ for a.*

$\qquad\qquad\quad \frac{5 \cdot 1}{8 \cdot 3}$ $\qquad$ *Multiply.*

$\qquad\qquad\quad \frac{5}{24}$ $\qquad$ *The answer is in simplest form.*

Helpful Hint

You can look for a common factor in a numerator and a denominator to determine whether you can simplify before multiplying.

B $a = \frac{9}{10}$ $\qquad a \cdot \frac{1}{3}$

$\qquad\qquad\quad \frac{9}{10} \cdot \frac{1}{3}$ $\qquad$ *Substitute $\frac{9}{10}$ for a.*

$\qquad\qquad\quad \frac{\overset{3}{\cancel{9}}}{10} \cdot \frac{1}{\underset{1}{\cancel{3}}}$ $\qquad$ *Use the GCF to simplify.*

$\qquad\qquad\quad \frac{3 \cdot 1}{10 \cdot 1}$ $\qquad$ *Multiply.*

$\qquad\qquad\quad \frac{3}{10}$ $\qquad$ *The answer is in simplest form.*

Multiply. Write each answer in simplest form.

C $\frac{3}{8} \cdot \frac{2}{9}$

$\qquad \frac{3}{8} \cdot \frac{2}{9} = \frac{3 \cdot 2}{8 \cdot 9}$ $\qquad$ *Multiply numerators. Multiply denominators.*

$\qquad\qquad\quad = \frac{6}{72}$ $\qquad$ *The GCF of 6 and 72 is 6.*

$\qquad\qquad\quad = \frac{1}{12}$ $\qquad$ *The answer is in simplest form.*

Multiplying Mixed Numbers

Janice and Carla are making homemade pasta from a recipe that calls for $1\frac{1}{2}$ cups of flour. They want to make $\frac{1}{3}$ of the recipe.

You can find $\frac{1}{3}$ of $1\frac{1}{2}$, or multiply $\frac{1}{3}$ by $1\frac{1}{2}$, to find how much flour Janice and Carla need.

1 Multiplying Fractions and Mixed Numbers

Multiply. Write each answer in simplest form.

A $\frac{1}{3} \cdot 1\frac{1}{2}$

$\frac{1}{3} \cdot \frac{3}{2}$ *Write $1\frac{1}{2}$ as an improper fraction. $1\frac{1}{2} = \frac{3}{2}$*

$\frac{1 \cdot 3}{3 \cdot 2}$ *Multiply numerators. Multiply denominators.*

$\frac{3}{6}$

$\frac{1}{2}$ *Write the answer in simplest form.*

B $1\frac{1}{5} \cdot \frac{2}{3}$

$\frac{6}{5} \cdot \frac{2}{3}$ *Write $1\frac{1}{5}$ as an improper fraction. $1\frac{1}{5} = \frac{6}{5}$*

$\frac{6 \cdot 2}{5 \cdot 3}$ *Multiply numerators. Multiply denominators.*

$\frac{12}{15}$

$\frac{4}{5}$ *Write the answer in simplest form.*

C $\frac{3}{4} \cdot 2\frac{1}{3}$

$\frac{3}{4} \cdot \frac{7}{3}$ Write $2\frac{1}{3}$ as an improper fraction. $2\frac{1}{3} = \frac{7}{3}$

$\frac{3}{4} \cdot \frac{7}{3}$ Use the GCF to simplify before multiplying.

$\frac{1 \cdot 7}{4 \cdot 1}$

$\frac{7}{4} = 1\frac{3}{4}$ You can write the answer as a mixed number.

2 Multiplying Mixed Numbers

Find each product. Write the answer in simplest form.

A $2\frac{1}{2} \cdot 1\frac{1}{3}$

$\frac{5}{2} \cdot \frac{4}{3}$ Write the mixed numbers as improper fractions. $2\frac{1}{2} = \frac{5}{2}$ $1\frac{1}{3} = \frac{4}{3}$

$\frac{5 \cdot 4}{2 \cdot 3}$ Multiply numerators.
Multiply denominators.

$\frac{20}{6}$

$3\frac{2}{6}$ Write the improper fraction as a mixed number.

$3\frac{1}{3}$ Simplify.

B $1\frac{1}{4} \cdot 1\frac{1}{3}$

$\frac{5}{4} \cdot \frac{4}{3}$ Write the mixed numbers as improper fractions. $1\frac{1}{4} = \frac{5}{4}$ $1\frac{1}{3} = \frac{4}{3}$

$\frac{5}{4} \cdot \frac{4}{3}$ Use the GCF to simplify before multiplying.

$\frac{5 \cdot 1}{1 \cdot 3}$ Multiply numerators. Multiply denominators.

$\frac{5}{3}$

$1\frac{2}{3}$ Write the answer as a mixed number.

C $5 \cdot 3\frac{2}{11}$

$5 \cdot 3\frac{2}{11}$

$5 \cdot \left(3 + \frac{2}{11}\right)$

$(5 \cdot 3) + \left(5 \cdot \frac{2}{11}\right)$ *Use the Distributive Property.*

$(5 \cdot 3) + \left(\frac{5}{1} \cdot \frac{2}{11}\right)$

$15 + \frac{10}{11}$ *Multiply.*

$15\frac{10}{11}$ *Add.*

Dividing Fractions and Mixed Numbers

Vocabulary

reciprocal

multiplicative inverse

Curtis is making sushi rolls. First, he will place a sheet of seaweed, called *nori,* on the sushi rolling mat. Then, he will use the mat to roll up rice, cucumber, avocado, and crabmeat. Finally, he will slice the roll into smaller pieces.

Curtis has 2 cups of rice and will use $\frac{1}{3}$ cup for each sushi roll. How many sushi rolls can he make?

Think: How many $\frac{1}{3}$ pieces equal 2 wholes?

There are six $\frac{1}{3}$ pieces in 2 wholes, so Curtis can make 6 sushi rolls.

Reciprocals can help you divide by fractions. Two numbers are **reciprocals** or *multiplicative inverses,* if their product is 1.

1			1		
$\frac{1}{3}$	$\frac{1}{3}$	$\frac{1}{3}$	$\frac{1}{3}$	$\frac{1}{3}$	$\frac{1}{3}$

Multiplicative Inverse Property	
Words	**Numbers**
The product of a nonzero number and its reciprocal, or **multiplicative inverse**, is 1.	$\frac{3}{4} \cdot \frac{4}{3} = 1$

EXAMPLE 1 **Finding Reciprocals**

Find the reciprocal.

A $\frac{1}{5}$

$\frac{1}{5} \cdot \blacksquare = 1$ *Think: $\frac{1}{5}$ of what number is 1?*

$\frac{1}{5} \cdot 5 = 1$ *$\frac{1}{5}$ of $\frac{5}{1}$ is 1.*

The reciprocal of $\frac{1}{5}$ is 5.

B $2\frac{1}{3}$

$\frac{7}{3} \cdot \blacksquare = 1$ *Write $2\frac{1}{3}$ as $\frac{7}{3}$.*

$\frac{7}{3} \cdot \frac{3}{7} = \frac{21}{21} = 1$ *$\frac{7}{3}$ of $\frac{3}{7}$ is 1.*

The reciprocal of $\frac{7}{3}$ is $\frac{3}{7}$.

Look at the relationship between the fractions $\frac{3}{4}$ and $\frac{4}{3}$. If you switch the numerator and denominator of a fraction, you will find its reciprocal. Dividing by a number is the same as multiplying by its reciprocal.

$$24 \div 4 = 6 \qquad 24 \cdot \frac{1}{4} = 6$$

2 Using Reciprocals to Divide Fractions and Mixed Numbers

Divide. Write each answer in simplest form.

A $\frac{4}{5} \div 5$

$\frac{4}{5} \div 5 = \frac{4}{5} \cdot \frac{1}{5}$ *Rewrite as multiplication using the reciprocal of 5, $\frac{1}{5}$.*

$= \frac{4 \cdot 1}{5 \cdot 5}$ *Multiply by the reciprocal.*

$= \frac{4}{25}$ *The answer is in simplest form.*

B $\frac{3}{4} \div \frac{1}{2}$

$\frac{3}{4} \div \frac{1}{2} = \frac{3}{4} \cdot \frac{2}{1}$ *Rewrite as multiplication using the reciprocal of $\frac{1}{2}$, $\frac{2}{1}$.*

$= \frac{3 \cdot \overset{1}{\cancel{2}}}{\underset{2}{\cancel{4}} \cdot 1}$ *Simplify before multiplying.*

$= \frac{3}{2}$ *Multiply.*

$= 1\frac{1}{2}$ *You can write the answer as a mixed number.*

C $2\frac{2}{3} \div 1\frac{1}{6}$

$2\frac{2}{3} \div 1\frac{1}{6} = \frac{8}{3} \div \frac{7}{6}$ *Write the mixed numbers as improper fractions. $2\frac{2}{3} = \frac{8}{3}$ and $1\frac{1}{6} = \frac{7}{6}$*

$= \frac{8}{3} \cdot \frac{6}{7}$ *Rewrite as multiplication.*

$= \frac{8 \cdot \overset{2}{\cancel{6}}}{\underset{1}{\cancel{3}} \cdot 7}$ *Simplify before multiplying.*

$= \frac{16}{7}$ *Multiply.*

$= 2\frac{2}{7}$ *You can write the answer as a mixed number.*

QUIZ

Multiply. Write each answer in simplest form.

1. $8 \cdot \frac{1}{9}$ **2.** $2 \cdot \frac{1}{5}$ **3.** $12 \cdot \frac{1}{4}$ **4.** $7 \cdot \frac{4}{9}$

5. $3 \cdot \frac{1}{7}$ **6.** $4 \cdot \frac{2}{11}$ **7.** $8 \cdot \frac{3}{4}$ **8.** $18 \cdot \frac{1}{3}$

1 Multiply. Write each answer in simplest form.

1. $\frac{1}{2} \cdot \frac{1}{3}$ **2.** $\frac{2}{5} \cdot \frac{1}{4}$ **3.** $\frac{4}{7} \cdot \frac{3}{4}$ **4.** $\frac{5}{6} \cdot \frac{3}{5}$

2 Evaluate the expression $b \cdot \frac{1}{5}$ for each value of b. Write the answer in simplest form.

5. $b = \frac{2}{3}$ **6.** $b = \frac{5}{8}$ **7.** $b = \frac{1}{4}$ **8.** $b = \frac{3}{5}$

1 Multiply. Write each answer in simplest form.

1. $1\frac{1}{4} \cdot \frac{2}{3}$ **2.** $2\frac{2}{3} \cdot \frac{1}{4}$ **3.** $\frac{3}{7} \cdot 1\frac{5}{6}$

4. $1\frac{1}{3} \cdot \frac{6}{7}$ **5.** $\frac{2}{3} \cdot 1\frac{3}{10}$ **6.** $2\frac{6}{11} \cdot \frac{2}{7}$

2 Find each product. Write the answer in simplest form.

7. $1\frac{5}{6} \cdot 1\frac{1}{8}$ **8.** $2\frac{2}{5} \cdot 1\frac{1}{12}$ **9.** $4 \cdot 5\frac{3}{7}$

10. $2\frac{3}{4} \cdot 1\frac{5}{6}$ **11.** $2\frac{3}{8} \cdot 5\frac{1}{5}$ **12.** $10\frac{1}{2} \cdot 1\frac{1}{4}$

1 Find the reciprocal.

1. $\frac{2}{7}$ **2.** $\frac{5}{9}$ **3.** $\frac{1}{9}$ **4.** $\frac{3}{11}$ **5.** $2\frac{3}{5}$

2 Divide. Write each answer in simplest form.

6. $\frac{5}{6} \div 3$ **7.** $2\frac{1}{7} \div 1\frac{1}{4}$ **8.** $\frac{5}{12} \div 5$ **9.** $1\frac{5}{8} \div \frac{3}{4}$

10. $\frac{2}{3} \div \frac{1}{6}$ **11.** $\frac{3}{10} \div 1\frac{2}{3}$ **12.** $\frac{4}{7} \div 1\frac{1}{7}$ **13.** $4 \div \frac{7}{8}$

To solve math problems, you need to know the basic mathematics before you can start applying it.

Catherine Asaro

WORKS CITED

HOLT McDOUGAL

Florida

Mathematics
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Jennie M. Bennett
Edward B. Burger
David J. Chard
Earlene J. Hall
Paul A. Kennedy
Freddie L. Renfro
Tom W. Roby
Janet K. Scheer
Cathy L. Seeley
Bert K. Waits